MATH BUSINESS/CALCULUS ~ 1325

Business Relations

Interest = (principal)(rate)(time)

Total cost = variable cost + fixed cost

Average cost per unit $= \frac{\text{total cost}}{\text{quanity}}$

Total revenue = (price per unit)(number of units sold)

Profit = total revenue – total cost

Compound Interest Formula

$S = P(1+r)^n$

$P = S(1+r)^{-n}$

$r_e = \left(1 + \frac{r}{n}\right)^n - 1$

$S = Pe^{rt}$ (continuous interest)

$P = Se^{-rt}$ (continuous interest)

$R_e = e^r - 1$ (continuous interest)

Ordinary Annuity Formulas

$$A = R\frac{1 - (1+r)^{-n}}{r}$$

$$S = R\frac{(1+r)^n - 1}{r}$$

Definition of Derivative of f(x)

$$f'(x) = \frac{d}{dx}\big(f(x)\big) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$$

The Derivative as a Rate of Change

$\frac{ds}{dt} = velocity\ at\ time\ t, where\ s\ is\ a\ position\ at\ time\ t$

$\frac{dr}{dq}$ *is called marginal revenue where* $r = pq$

$\frac{dc}{dq}$ *is called marginal cost, if* $c = f(q)$ *is total cost function of* q *units*

The relative rate of change of f(x) is $\frac{f'(x)}{f(x)}$

The percentage rate of change is $\frac{f'(x)}{f(x)} * 100\%$

Elasticity for Demand q = q (p)

$$\eta = \frac{p}{q} \cdot \frac{dq}{dp} = \frac{\frac{p}{q}}{\frac{dp}{dq}}$$

$|\eta| > 1$ *elastic*

$|\eta| = 1$ *unit elasticity*

$|\eta| < 1$ *inelastic*

Average cost per unit

$$\bar{c} = \frac{c}{q}$$

Consumers' Surplus for Demand p = f(q)

$$CS = \int_0^{q_0} [f(q) - p_0]\,dq$$

Producers' Surplus for Supply p = g(q)

$$PS = \int_0^{q_0} [p_0 - g(q)]\,dq$$

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Increasing/Decreasing – Concave Up/ Concave Down

Critical Points: $x = c$ is a critical point of $f(x)$ provided either: $f'(c) = 0$ or $f'(c)$ does not exist.

Increasing/Decreasing

1. If $f'(x) > 0$ for all x in an interval I then $f(x)$ is increasing on the interval I.
2. If $f'(x) < 0$ for all x in an interval I then $f(x)$ is decreasing on the interval I.
3. If $f'(x) = 0$ for all x in an interval I then $f(x)$ is constant on the interval I.

Concave Up/Concave Down

1. If $f''(x) > 0$ for all x in an interval I then $f(x)$ is concave up on the interval I.
2. If $f''(x) < 0$ for all x in an interval I then $f(x)$ is concave down on the interval I.

Inflection Points

X = c is a inflection point of $f(x)$ if the concavity changes at $x = c$

Relative Extrema

Definition:

1. $x = c$ is a relative maximum of $f(x)$ if $f(c) \geq f(x)$ for all x near c.
2. $x = c$ is a relative minimum of $f(x)$ if $f(c) \leq f(x)$ for all x near c.

1st Derivative Test:

If $x = c$ is a critical point of $f(x)$ then $x = c$ is

1. a rel. max of $f(x)$ if $f'(x) > 0$ to the left of $x = c$ and $f'(x) < 0$ to the right of $x = c$
2. a rel. min of $f(x)$ if $f'(x) < 0$ to the left of $x = c$ and $f'(x) > 0$ to the right of $x = c$
3. not a relative extrema of $f(x)$ if $f'(x)$ is the same sign on both sides of $x = c$

2nd Derivative Test:

If $x = c$ is a critical point of $f(x)$ such that $f'(c) = 0$ then $x = c$

1. a relative maximum of $f(x)$ if $f''(c) < 0$
2. a relative minimum of $f(x)$ if $f''(c) > 0$
3. may be a relative maximum, relative minimum or neither if $f''(c) = 0$

Mean Value Theorem

If $f(x)$ is continuous on the closed interval [a, b] and differentiable on the open interval (a,b) then there is a number $a < c < b$ such that $f'(c) = \frac{f(b)-f(a)}{b-a}$

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